

$$\begin{aligned}
 (1) \quad \sum_{n=0}^{\infty} \frac{2n+1}{(2n)!} &= 1 + \sum_{n=1}^{\infty} \left( \frac{1}{(2n-1)!} + \frac{1}{(2n)!} \right) \\
 &= 1 + \left( \frac{1}{1!} + \frac{1}{2!} \right) + \left( \frac{1}{3!} + \frac{1}{4!} \right) + \dots \\
 &= e
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad \sum_{n=0}^{\infty} \frac{n+1}{(2n+1)!} &= \frac{1}{2} \sum_{n=0}^{\infty} \frac{(2n+1)+1}{(2n+1)!} \\
 &= \frac{1}{2} \sum_{n=0}^{\infty} \left( \frac{1}{(2n)!} + \frac{1}{(2n+1)!} \right) \\
 &= \frac{1}{2} \left\{ \left( \frac{1}{0!} + \frac{1}{1!} \right) + \left( \frac{1}{2!} + \frac{1}{3!} \right) + \dots \right\} \\
 &= \frac{e}{2}
 \end{aligned}$$

$$(3) \quad \sum_{n=1}^{\infty} \frac{2n}{(2n+1)!}$$

$$\begin{aligned}
 (4) \quad \sum_{n=1}^{\infty} \frac{n}{(n+1)!} &= \sum_{n=1}^{\infty} \frac{(n+1)-1}{(n+1)!} \\
 &= \sum_{n=1}^{\infty} \left\{ \frac{1}{n!} - \frac{1}{(n+1)!} \right\} \\
 &= \lim_{n \rightarrow \infty} \left\{ 1 - \frac{1}{(n+1)!} \right\} \\
 &= 1
 \end{aligned}$$