

問題

$\sqrt{2} + \sqrt{3} > \pi$ を証明せよ。

解答

$$\pi < 3 + \frac{1}{7} = \frac{22}{7} = \frac{880}{280}$$

$$\sqrt{2} - 1 < \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}}} = \frac{29}{70} = \frac{116}{280}$$

$$\sqrt{3} + 1 < 2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1}}}}}}} = \frac{153}{56} = \frac{765}{280}$$

より、

$$\pi < \frac{22}{7} = \frac{880}{280} < \sqrt{2} + \sqrt{3} < \frac{29}{70} + \frac{153}{56} = \frac{116}{280} + \frac{765}{280} = \frac{881}{280} \quad \therefore \quad \pi < \frac{880}{280} < \sqrt{2} + \sqrt{3} < \frac{881}{280}$$

$$\therefore \sqrt{2} + \sqrt{3} > \pi$$