● 問題 412〈追加問題〉解答＜三角定規＞

$$a=2\sin 50^\circ \quad =2\cos 40^\circ \quad =2\cos\frac{2\pi}{9}$$

だから，$a$ は方程式　$x^3-3x+1=0$　の解で

$$a^3-3a+1=0 \quad \cdots①$$

$$\frac{1}{a^2-a-2}-\frac{1}{a^2+a-2}+\frac{1}{a^2-2a+1}-\frac{1}{a^2+2a+1}$$

$$=\frac{1}{(a-2)(a+1)}-\frac{1}{(a+2)(a-1)}+\frac{1}{(a-1)^2}-\frac{1}{(a+1)^2}$$

$$=\frac{1}{a+1}\left(\frac{1}{a-2}-\frac{1}{a+1}\right)+\frac{1}{a-1}\left(\frac{1}{a-1}-\frac{1}{a+2}\right)$$

$$=\frac{1}{a+1}\cdot\frac{3}{(a-2)(a+1)}+\frac{1}{a-1}\cdot\frac{3}{(a-1)(a+2)}$$

$$=\frac{3}{a^3-3a-2}+\frac{3}{a^3-3a+2}$$

$$=\frac{3}{-1-2}+\frac{3}{-1+2} \quad (\because ①)$$

$$=-1+3=\mathbf{2} \quad \cdots[答]$$